BU-661-M - July 1978

Abstract

Comments on computer output generated at the University of Hohenheim, July 1977, using Harvey's program on five small sets of hypothetical data.

BU-661-M S.R.S.

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                    LISTING OF PARAMETER CARDS  ON PROBLEM NO.  1

IJOB NAB  NCD  ICN1 NLHM NRHM NMEA NME NNEA  NAF  N2F  NPR  NLC NCPR IRAN MPOP LIOP  IN    FILENAME
 1    2    1    0    7    1    0    2    0    0    0    0    0    0    0    1    3    5

FOR I=   1  MEN(I)   LIT(I)   NCL(I)   MPOL   LME(I)   IREG(I)
              1      WOMEN      4       0       1        1

IDEN(J) WHERE J=K2 BY STEPS OF ONE AS LONG AS J IS LESS THAN K2+NCL(I)+1
   1         2        3   4

FOR I=   2  MEN(I)   LIT(I)   NCL(I)   MPOL   LME(I)   IREG(I)
              2      MEN        4       0       1        2

IDEN(J) WHERE J=K2 BY STEPS OF ONE AS LONG AS J IS LESS THAN K2+NCL(I)+1
   1         2        3   4

FOR I= 1  NEGY(I) LNY(I) LHY(I) KBEF(I) NDECY(I) YM(I) LITY(I)
             0      0      2       3        0    0.00000 SCORE
```

← Why no opportunity to use one's own problem name? [1]

Problem No. 1 [2]

Exercise 7.10, LM page 328. *
Solutions Manual, pages 101-102.

8	-	9	10
13	-	-	-
-	6	14	-
12	14	10	24

COMMENTS ON COMPUTER OUTPUT

Comments on computer output generated at the University of Hohenheim, July 1977, using Harvey's program on five small sets of hypothetical data.

S. R. Searle

Biometrics Unit, Cornell University, Ithaca, New York

* References:

LM: "Linear Models", S. R. Searle, Wiley, 1971.

Solutions Manual: "Solutions Manual to LM", S. R. Searle, Biometrics Unit, Cornell University, 1976.

BU-417-M: "Using the R( ) notation ···", S. R. Searle, Paper BU-417-M, Biometrics Unit, Cornell University, 1972.

BU-608-M: "Illustrative calculations of sums of squares ···", S. R. Searle, Paper BU-608-M, Biometrics Unit, Cornell University, 1977.

LISTING OF X MATRIX FOR PROBLEM NO. 1

1.00	1.00	0.00	0.00	1.00	0.00	0.00	8.00
1.00	1.00	0.00	0.00	0.00	0.00	1.00	9.00
1.00	1.00	0.00	0.00	-1.00	-1.00	-1.00	10.00
1.00	0.00	1.00	0.00	1.00	0.00	0.00	13.00
1.00	0.00	0.00	1.00	0.00	1.00	0.00	6.00
1.00	0.00	0.00	1.00	0.00	0.00	1.00	14.00
1.00	-1.00	-1.00	-1.00	1.00	0.00	0.00	12.00
1.00	-1.00	-1.00	-1.00	0.00	1.00	0.00	14.00
1.00	-1.00	-1.00	-1.00	0.00	0.00	1.00	10.00
1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	24.00

This is not an $\underset{\sim}{X}$-matrix of the model $E(\underset{\sim}{y}) = \underset{\sim\sim}{Xb}$. It is $\underset{\sim}{X}$ and $\underset{\sim}{y}$: the last column is $\underset{\sim}{y}$, and the other columns are the $\underset{\sim}{X}$-matrix for the restricted model $E(\underset{\sim}{y}) = \underset{\sim\sim}{Xb}$ with Σ-restrictions. [3]

(See BU-608-M)

The model for this example is $y_{ij} = \mu + \alpha_i + \beta_j + e_{ij}$ for $i = 1, \cdots, 4$ and $j = 1, \cdots, 4$. The $\underset{\sim}{X}$-matrix for this, $\underset{\sim}{X}_S$ say, is [4]

$$
\underset{\sim}{X}_S = \begin{bmatrix}
1 & 1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\
1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\
1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\
1 & \cdot & 1 & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\
1 & \cdot & \cdot & 1 & \cdot & \cdot & 1 & \cdot & \cdot \\
1 & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & 1 & \cdot \\
1 & \cdot & \cdot & \cdot & 1 & 1 & \cdot & \cdot & \cdot \\
1 & \cdot & \cdot & \cdot & 1 & \cdot & 1 & \cdot & \cdot \\
1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & 1 & \cdot \\
1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & 1
\end{bmatrix}.
$$

The program's $\underset{\sim}{X}$-matrix is for this model but with restrictions $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 0$ and $\beta_1 + \beta_2 + \beta_3 + \beta_4 = 0$. Apply these to $\underset{\sim}{b}$ corresponding to $\underset{\sim}{X}_S$ and the result is the first 7 columns of the above output.

This is misleading. Absorbing equations is only an arithmetical tool. [5]

TOTAL LEAST-SQUARES ANALYSIS. NO EQUATIONS ABSORBED. RE=NO. CARDS= 10

DISTRIBUTION OF CLASS AND SUBCLASS NUMBERS FOR PROBLEM NO. 1

This means: Numbers of observations per class and subclass. [6]

IDENTIFICATION		NO.	
WOMEN	1	3	$n_{i.}$
WOMEN	2	1	
WOMEN	3	2	
WOMEN	4	4	
MEN	1	3	$n_{.j}$
MEN	2	2	
MEN	3	3	
MEN	4	2	

OVERALL MEANS AND STANDARD DEVIATIONS OF LHM FOR PROBLEM NO. 1 — "Left-hand member", presumably. Of what? [7]

CODED LHM	INDEPENDENT VARIABLES		MEAN	S.D.
1	MU		1.000[illegible]	[illegible]
2	WOMEN	1	-0.1[illegible]	0.875[illegible]
3	WOMEN	2	-[illegible]	[illegible].474[illegible]
4	WOMEN	3	-0.200[illegible]	[illegible].788[illegible]1
5	MEN	1	[illegible]	[illegible].7[illegible]
6	MEN	2	[illegible]000	0.66667
7	MEN	3	[illegible].1[illegible]	.7379[illegible]

These values have no use in design models.

These entries are means of the entries in the columns of the $\underset{\sim}{X}$-matrix; [8]

e.g., $1 = \frac{1}{10}(1 + 1 + \cdots \text{ ten } 1\text{'s})$

$-.1 = \frac{1}{10}(1 + 1 + 1 + 0 + 0 + 0 - 1 - 1 - 1 - 1).$

These entries are the sample s.d.'s not of the means but of the "observation" in each column; [9]

e.g., $.87560 = \sqrt{\dfrac{3(1)^2 + 3(0)^2 + 4(-1)^2 - 10(.1)^2}{9}}$.

OVERALL MEANS AND STANDARD DEVIATIONS OF RHM ← i.e. of $\underset{\sim}{y}$

SCORE MEAN= 12.[illegible] ← $\bar{y}$ S.D.= 4.[illegible]6655 ← S.D. of y, not of $\bar{y}$. [10]

SUMS OF SQUARES, C.P. AND CORRELATIONS AMONG LHM FOR PROBLEM NO. 1

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW		COLUMN		S.SQS. OR C.P.	CORRELATION
1	1	MU		MU		10.[illegible]	1.[illegible]
1	2	MU		WOMEN	1	-1.00000000	0.0000
1	3	MU		WOMEN	2	-3.[illegible]	0.0000
1	4	MU		WOMEN	3	-2.[illegible]	0.0000
1	5	MU		MEN	1	1.[illegible]	0.0000
1	6	MU		MEN	2	0.[illegible]	0.0000
1	7	MU		MEN	3	1.0000000[illegible]	[illegible]
2	2	WOMEN	1	WOMEN	1	7.[illegible]	1.0000
2	3	WOMEN	1	WOMEN	2	4.[illegible]	0.6956
2	4	WOMEN	1	WOMEN	3	4.[illegible]	0.6113
2	5	WOMEN	1	MEN	1	0.00000000	0.0172
2	6	WOMEN	1	MEN	2	-1.00000000	-0.1903
2	7	WOMEN	1	MEN	3	0.00000000	0.0172
3	3	WOMEN	2	WOMEN	2	5.00000000	1.0000
3	4	WOMEN	2	WOMEN	3	4.[illegible]	0.7[illegible]96
3	5	WOMEN	2	MEN	1	1.[illegible]	0.290[illegible]
3	6	WOMEN	2	MEN	2	[illegible]	[illegible]
3	7	WOMEN	2	MEN	3	0.00000000	0.0669
4	4	WOMEN	3	WOMEN	3	6.00000000	1.0000
4	5	WOMEN	3	MEN	1	0.[illegible]	0.0382
4	6	WOMEN	3	MEN	2	1.00000000	0.2113
4	7	WOMEN	3	MEN	3	1.00000000	0.2291
5	5	MEN	1	MEN	1	5.00000000	1.0000
5	6	MEN	1	MEN	2	2.00000000	0.4518
5	7	MEN	1	MEN	3	2.00000000	0.3[illegible]
6	6	MEN	2	MEN	2	4.00000000	1.0000
6	7	MEN	2	MEN	3	2.00000000	0.4518
7	7	MEN	3	MEN	3	5.00000000	1.0000

These values have no use in design models.

These are s/s and s/cp of the "x"'s – i.e., X'X of the printed X, with its last column (which is $\underset{\sim}{y}$) omitted. In regression, these would be uncorrected s/s and s/cp. [11]

SUMS OF CROSSPRODUCTS AND CORRELATIONS OF LHM WITH RHM FOR PROBLEM NO. 1

RHM	LHM	RHM NAME	INDEPENDENT VARIABLE		C.P.	CORRELATION
1	1	SCORE	MU		120.00000000	0.0000
1	2	SCORE	WOMEN	1	-33.00000000	-0.5366
1	3	SCORE	WOMEN	2	-47.00000000	-0.3646
1	4	SCORE	WOMEN	3	-40.00000000	-0.4538
1	5	SCORE	MEN	1	-1.00000000	-0.3942
1	6	SCORE	MEN	2	-14.00000000	-0.4698
1	7	SCORE	MEN	3	-1.00000000	-0.3942

$\underset{\sim}{X}'\underset{\sim}{y}$ for the printed $\underset{\sim}{X}$ [12]

INVERSE ELEMENTS FOR PROBLEM NO. 1

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW		COLUMN		INVERSE ELEMENT FIXED POINT FORMAT	FLOATING POINT FORMAT
1	1	MU		MU		0.14914773	0.14914773E 00
1	2	MU		WOMEN	1	-0.05255682	-0.52556818E-01
1	3	MU		WOMEN	2	0.18039773	0.18039773E 00
1	4	MU		WOMEN	3	-0.04119318	-0.41193182E-01
1	5	MU		MEN	1	-0.07954545	-0.79545455E-01
1	6	MU		MEN	2	0.03977273	0.39772727E-01
1	7	MU		MEN	3	-0.00568182	-0.56818182E-02
2	2	WOMEN	1	WOMEN	1	0.34232955	0.34232955E 00
2	3	WOMEN	1	WOMEN	2	-0.20880682	-0.20880682E 00
2	4	WOMEN	1	WOMEN	3	-0.12357955	-0.12357955E 00
2	5	WOMEN	1	MEN	1	0.01136364	0.11363636E-01
2	6	WOMEN	1	MEN	2	0.11931818	0.11931818E 00
2	7	WOMEN	1	MEN	3	-0.01704545	-0.17045455E-01
3	3	WOMEN	2	WOMEN	2	0.89914773	0.89914773E 00
3	4	WOMEN	2	WOMEN	3	-0.44744318	-0.44744318E 00
3	5	WOMEN	2	MEN	1	-0.32954545	-0.32954545E 00
3	6	WOMEN	2	MEN	2	0.16477273	0.16477273E 00
3	7	WOMEN	2	MEN	3	0.11931818	0.11931818E 00
4	4	WOMEN	3	WOMEN	3	0.59232955	0.59232955E 00
4	5	WOMEN	3	MEN	1	0.23863636	0.23863636E 00
4	6	WOMEN	3	MEN	2	-0.24431818	-0.24431818E 00
4	7	WOMEN	3	MEN	3	-0.10795455	-0.10795455E 00
5	5	MEN	1	MEN	1	0.40909091	0.40909091E 00
5	6	MEN	1	MEN	2	-0.20454545	-0.20454545E 00
5	7	MEN	1	MEN	3	-0.11363636	-0.11363636E 00
6	6	MEN	2	MEN	2	0.47727273	0.47727273E 00
6	7	MEN	2	MEN	3	-0.06818182	-0.68181818E-01
7	7	MEN	3	MEN	3	0.29545455	0.29545455E 00

Presumably this is $(\underset{\sim}{X}'\underset{\sim}{X})^{-1}$ of the printed $\underset{\sim}{X}$ (omitting its last column, $\underset{\sim}{y}$). [13]

See comment [20]

THE DETERMINANT OF THE CORRELATION MATRIX IS 0.0536380952366926 0.536380952366926E-01

SCORE	1	MU		12.32954545	0.12329545E 02
SCORE	2	WOMEN	1	-4.51136364	-0.45113636E 01
SCORE	3	WOMEN	2	2.07954546	0.20795455E 01
SCORE	4	WOMEN	3	-0.23863636	-0.23863636E 00
SCORE	5	MEN	1	-1.40909091	-0.14090909E 01
SCORE	6	MEN	2	-3.54545455	-0.35454545E 01
SCORE	7	MEN	3	-0.63636364	-0.63636364E 00

No title. What are these values? [14]
Presumably solutions (see below).

What constants? This is vague. Standard errors of what? [15]

LISTING OF CONSTANTS, LEAST-SQUARES MEANS AND STANDARD ERRORS FOR PROBLEM NO. 1

RHM NAME	ROW CODE	INDEPENDENT VARIABLE		NO. OBS.	CONSTANT ESTIMATE (A)		LEAST-SQUARES MEAN (B)	STANDARD ERROR (C)
SCORE	1	MU		10	12.32954545		12.32954545	1.86671724
SCORE	2	WOMEN	1	3	-4.51136364	add to zero	7.81818182	3.00447325
SCORE	3	WOMEN	2	1	2.07954546		14.40909091	5.73772405
SCORE	4	WOMEN	3	2	-0.23863636		12.09090909	3.92412542
SCORE	5	WOMEN	4	4	2.67045455		15.00000000	2.41679728
SCORE	5	MEN	1	3	-1.40909091	add to zero	10.92045455	3.05377510
SCORE	6	MEN	2	2	-3.54545455		8.78409091	4.06127716
SCORE	7	MEN	3	3	-0.63636364		11.69318182	3.18151378
SCORE	8	MEN	4	2	5.59090909		17.92045455	[illegible]50441285

(A), (B), (C), see below

[illegible]

LISTING OF INVERSE ELEMENTS OF SEGMENTS FOR PROBLEM NO. 1

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW		INDEPENDENT VARIABLES COL		INVERSE ELEMENT FIXED POINT FORMAT	INVERSE ELEMENT FLOATING POINT FORMAT
1	1	MU		MU		6.70476190	0.67047619E 01
2	2	WOMEN	1	WOMEN	1	6.50000000	0.65000000E 01
2	3	WOMEN	1	WOMEN	2	3.50000000	0.35000000E 01
2	4	WOMEN	1	WOMEN	3	4.00000000	0.40000000E 01
3	3	WOMEN	2	WOMEN	2	3.66666667	0.36666667E 01
3	4	WOMEN	2	WOMEN	3	3.50000000	0.35000000E 01
4	4	WOMEN	3	WOMEN	3	5.16666667	0.51666667E 01
5	5	MEN	1	MEN	1	4.00000000	0.40000000E 01
5	6	MEN	1	MEN	2	2.00000000	0.20000000E 01
5	7	MEN	1	MEN	3	2.00000000	0.20000000E 01
6	6	MEN	2	MEN	2	3.16666667	0.31666667E 01
6	7	MEN	2	MEN	3	1.50000000	0.15000000E 01
7	7	MEN	3	MEN	3	4.50000000	0.45000000E 01

These values are presumably made available for use in the "invert part of the inverse" rule. See LM p. 120.

(A) A poor title. This column is a solution vector $\underset{\sim}{b}^o$. It is the solution vector in which $\Sigma\alpha^o = 0$ and $\Sigma\beta^o = 0$. These solutions are b.l.u.e.'s of the corresponding parameters in the restricted model with restrictions $\Sigma\alpha = 0$ and $\Sigma\beta = 0$. The solution vector given (partially) in the Solutions Manual, $\underset{\sim}{b}^o_S$ say, is [16]

$$\underset{\sim}{b}^o_S = (0 \quad 13\tfrac{9}{22} \quad 20 \quad 17\tfrac{15}{22} \quad 20\tfrac{13}{22} \quad -7 \quad -9\tfrac{3}{22} \quad -6\tfrac{5}{22} \quad 0)$$

$$= (0 \quad 13.41 \quad 20 \quad 17.68 \quad 20.59 \quad -7 \quad -9.13 \quad -6.23 \quad 0)$$

Note that b.l.u.e.'s of estimable functions in the unrestricted model are the same for both $\underset{\sim}{b}^o_S$ and $\underset{\sim}{b}^o$ given by the program. Examples are:

BLUE	Program $\underset{\sim}{b}^o$	$\underset{\sim}{b}^o_S$
$\alpha_1^o - \alpha_2^o$	$-4.51 - 2.08 = -6.59$	$13.41 - 20 = -6.59$
$\beta_1^o - \beta_2^o$	$-1.41 - (-3.54) = 2.13$	$-7 - (-9.13) = 2.13$

(B) These values are μ^o, $\mu^o + \alpha_i^o$ and $\mu^o + \beta_j^o$. Now we know what is meant by "least squares mean": it is a linear combination of elements of the solution vector. But it is <u>not</u> a b.l.u.e. of an estimable function in the unrestricted model. In the unrestricted model the terms $(\mu + \alpha_i + \beta_j)$ are estimable, and no function of such terms reduces to $\mu + \alpha_i$ or to $\mu + \beta_j$; hence $\mu^o + \alpha_i^o$ and $\mu^o + \beta_j^o$ are not b.l.u.e.'s of estimable functions; and evidence of this is available from their values, both from the program output and the SRS solution. Examples are: [17]

	Program $\underset{\sim}{b}^o$	$\underset{\sim}{b}^o_S$
$\mu^o + \alpha_1^o$	$12.329 - 4.511 = 7.818$	$0 + 13.41 = 13.41 \neq 7.818$
$\mu^o + \beta_1^o$	$12.329 - 1.409 = 10.920$	$0 - 7 = 7 \neq 10.920$

In the restricted model, with restrictions $\Sigma\alpha_i = 0$ and $\Sigma\beta_j = 0$, $\mu^o + \alpha_i^o$ <u>is</u> a b.l.u.e. of $\mu + \alpha_i$. Note that for any row in which there is an observation in every column, $\mu^o + \alpha_i^o = \bar{y}_{i.}$. "Women 4" is an example. This is because in the unrestricted model $\bar{y}_{4.}$ is the b.l.u.e. of $\mu + \alpha_4 + \frac{1}{b}\sum_{j=1}^{b}\beta_j$ which reduces to $\mu + \alpha_4$ in the restricted model. Thus [18]

$$\widehat{\mu + \alpha_4 + \tfrac{1}{4}\Sigma\beta_j} = \mu^o + \alpha_4^o + \tfrac{1}{4}\Sigma\beta_j^o = 12.329 + 2.671 + \tfrac{1}{4}(0) = 15, \text{ from program output}$$

$$= 0 + 20\tfrac{13}{22} - \tfrac{1}{4}(7 + 9\tfrac{3}{22} + 6\tfrac{5}{22}) = 15, \text{ from } b_S^o$$

$$= \bar{y}_{4.} = 15, \text{ from data.}$$

Ⓒ These are s.e.'s of the entries in Ⓑ, i.e. of $\widehat{\mu + \alpha_i}$ and $\widehat{\mu + \beta_j}$ in the restricted model. But they are of <u>no</u> use for finding s.e.'s of differences $\widehat{\alpha_i - \alpha_{i'}}$ or $\widehat{\beta_j - \beta_{j'}}$. An example of a value is the second entry,

$$3.004 = \sqrt{\underbrace{[.1491 + .3423 - 2(.0526)]}_{\text{From the "inverse element" column alongside note [13].}}\underbrace{23.36}_{\hat{\sigma}_e^2 \text{ from the ANOVA above note [20].}}} \qquad [19]$$

This part of the title is unexplained.

LEAST-SQUARES ANALYSIS OF VARIANCE

SCORE

SOURCE		D.F.		SUM OF SQUARES	MEAN SQUARES	F
TOTAL		10	$y'y$	1662.000000		
TOTAL REDUCTION		7	$R(\mu,\alpha,\beta)$	1591.909091	227.415584	9.734
MU-YM		1		1019.242424	1019.242424	43.625
WOMEN	Misleading	3	$R(\alpha\|\mu,\beta)$	87.909091	29.303030	1.254
MEN	titles	3	$R(\beta\|\mu,\alpha)$	79.909091	26.636364	1.140
REMAINDER		3	SSE	70.090909	23.363636	

in view of their full explanations

This is <u>not</u> $R(\mu) = 120^2/10 = 1440$. It has been calculated as $(12.329)^2/.1491 = 1019.24$, which is $(\mu^o)^2$ for $\mu^o = 12.329$ of the program, divided by the entry in the "inverse element" column corresponding to (μ,μ), namely .1491. [20]

```
LISTING OF PARAMETER CARDS FOR PROBLEM NO. 2

IJOB NAB  ...  ICN1 NIHM NROW NVEK ... KNEA  ...  K2F  ...  NLC MORE THAN MORE LIMP  16     FILENAME
 2    1   ...                1                    1                   6    8   1

FOR I=  1  MEN(I)  LIT(I)  NCL(I)  MPOL  LMF(I)  IREF(I)
             1      ROWS      2                     1

IDEN(J) WHERE J=K2 BY STEPS OF ONE AS LONG AS J IS LESS THAN K2+NCL(I)+1
    1       2

FOR I=  2  MEN(I)  LIT(I)  NCL(I)  MPOL  LMF(I)  IREF(I)
             2     COLUMN                           2

IDEN(J) WHERE J=K2 BY STEPS OF ONE AS LONG AS J IS LESS THAN K2+NCL(I)+1
    1       2         3

FOR I=   1  INT1(I) INT2(I)  NMC(I)
              1       2

FOR I= 1  NEGY(I) LNY(I) IHY(I) KREF(I) NDECY(I) YM(I) LITY(I)
                           2       3       0     0.00000 Y
```

Problem No. 2

This is the all-cells filled example in paper BU-608-M.

	Data			Totals
	7,9	6	2	24
	8	4,8	12	32
Totals	24	18	14	56

[21]

It is important to remember that this example has <u>all</u> <u>cells</u> <u>filled</u>.

```
                   LISTING OF X MATRIX FOR PROBLEM NO. 2
  1.00    1.00    1.00    0.00    1.00    0.00    7.00
  1.00    1.00    1.00    0.00    1.00    0.00    9.00
  1.00    1.00    0.00    1.00    0.00    1.00    6.00
  1.00    1.00   -1.00   -1.00   -1.00   -1.00    2.00
  1.00   -1.00    1.00    0.00   -1.00    0.00    8.00
  1.00   -1.00    0.00    1.00    0.00   -1.00    4.00
  1.00   -1.00    0.00    1.00    0.00   -1.00    8.00
  1.00   -1.00   -1.00   -1.00    1.00    1.00   12.00
```

This is $\underset{\sim}{X}$ for a model with Σ-restrictions [22]

$$\Sigma\alpha_i = 0 \qquad \Sigma\beta_j = 0$$

$$\sum_i \gamma_{ij} = 0 \ \forall j \qquad \sum_j \gamma_{ij} = 0 \ \forall i \ .$$

The last column is $\underset{\sim}{y}$.

TOTAL LEAST-SQUARES ANALYSIS. NO EQUATIONS ABSORBED. DF=NO. CARDS= 8

DISTRIBUTION OF CLASS AND SUBCLASS NUMBERS FOR PROBLEM NO. 2

IDENTIFICATION				NO.
ROWS		1		4
ROWS		2		4
COLUMN		1		3
COLUMN		2		3
COLUMN		3		2
ROWS	X COLUMN	1	1	2
ROWS	X COLUMN	1	2	1
ROWS	X COLUMN	1	3	1
ROWS	X COLUMN	2	1	1
ROWS	X COLUMN	2	2	2
ROWS	X COLUMN	2	3	1

OVERALL MEANS AND STANDARD DEVIATIONS OF LHM FOR PROBLEM NO. 2

CODED LHM	INDEPENDENT VARIABLES				MEAN	S.D.
1	MU				1.00000	0.00000
2	ROWS		1		0.00000	1.06904
3	COLUMN		1		0.12500	0.83452
4	COLUMN		2		0.12500	0.83452
5	ROWS	COLUMN	1	1	0.12500	0.83452
6	ROWS	COLUMN	1	2	-0.12500	0.83452

Comments similar to those on Problem 1 apply here. [23]

OVERALL MEANS AND STANDARD DEVIATIONS OF RHM

Y MEAN= 7.00000 S.D.= 3.07060

SUMS OF SQUARES, C.P. AND CORRELATIONS AMONG LHM FOR PROBLEM NO. 2

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW				COLUMN				S.SQS. OR C.P.	CORRELATION
1	1	MU				MU				8.00000000	1.0000
1	2	MU				ROWS		1		0.00000000	0.0000
1	3	MU				COLUMN		1		1.00000000	0.0000
1	4	MU				COLUMN		2		1.00000000	0.0000
1	5	MU				ROWS	COLUMN	1	1	1.00000000	0.0000
1	6	MU				ROWS	COLUMN	1	2	-1.00000000	0.0000
2	2	ROWS		1		ROWS		1		8.00000000	1.0000
2	3	ROWS		1		COLUMN		1		1.00000000	0.1601
2	4	ROWS		1		COLUMN		2		-1.00000000	-0.1601
2	5	ROWS		1		ROWS	COLUMN	1	1	1.00000000	0.1601
2	6	ROWS		1		ROWS	COLUMN	1	2	1.00000000	0.1601
3	3	COLUMN		1		COLUMN		1		5.00000000	1.0000
3	4	COLUMN		1		COLUMN		2		2.00000000	0.3846
3	5	COLUMN		1		ROWS	COLUMN	1	1	1.00000000	0.1795
3	6	COLUMN		1		ROWS	COLUMN	1	2	0.00000000	0.0256
4	4	COLUMN		2		COLUMN		2		5.00000000	1.0000
4	5	COLUMN		2		ROWS	COLUMN	1	1	0.00000000	-0.0256
4	6	COLUMN		2		ROWS	COLUMN	1	2	-1.00000000	-0.1795
5	5	ROWS	COLUMN	1	1	ROWS	COLUMN	1	1	5.00000000	1.0000
5	6	ROWS	COLUMN	1	1	ROWS	COLUMN	1	2	2.00000000	0.4359
6	6	ROWS	COLUMN	1	2	ROWS	COLUMN	1	2	5.00000000	1.0000

SUMS OF CROSSPRODUCTS AND CORRELATIONS OF LHM WITH RHM FOR PROBLEM NO. 2

RHM	LHM	RHM NAME	INDEPENDENT VARIABLE				C.P.	CORRELATION
1	1	Y	MU				56.00000000	0.0000
1	2	Y	ROWS	1			-4.00000000	-0.3482
1	3	Y	COLUMN	1			10.00000000	0.1672
1	4	Y	COLUMN	2			4.00000000	-0.1672
1	5	Y	ROWS	COLUMN	1	1	18.00000000	0.6132
1	6	Y	ROWS	COLUMN	1	2	4.00000000	0.6132

LISTING OF INVERSE ELEMENTS FOR PROBLEM NO. 2

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW				COLUMN				INVERSE ELEMENT FIXED POINT FORMAT	FLOATING POINT FORMAT
1	1	MU				MU				0.13888889	0.13888889E 00
1	2	MU				ROWS	1			0.00000000	0.56843419E-13
1	3	MU				COLUMN	1			-0.01388889	-0.13888889E-01
1	4	MU				COLUMN	2			-0.01388889	-0.13888889E-01
1	5	MU				ROWS	COLUMN	1	1	-0.04166667	-0.41666667E-01
1	6	MU				ROWS	COLUMN	1	2	0.04166667	0.41666667E-01
2	2	ROWS	1			ROWS	1			0.13888889	0.13888889E 00
2	3	ROWS	1			COLUMN	1			-0.04166667	-0.41666667E-01
2	4	ROWS	1			COLUMN	2			0.04166667	0.41666667E-01
2	5	ROWS	1			ROWS	COLUMN	1	1	-0.01388889	-0.13888889E-01
2	6	ROWS	1			ROWS	COLUMN	1	2	-0.01388889	-0.13888889E-01
3	3	COLUMN	1			COLUMN	1			0.26388889	0.26388889E 00
3	4	COLUMN	1			COLUMN	2			-0.11111111	-0.11111111E 00
3	5	COLUMN	1			ROWS	COLUMN	1	1	-0.04166667	-0.41666667E-01
3	6	COLUMN	1			ROWS	COLUMN	1	2	-0.00000000	-0.24000555E-12
4	4	COLUMN	2			COLUMN	2			0.26388889	0.26388889E 00
4	5	COLUMN	2			ROWS	COLUMN	1	1	0.00000000	0.45474735E-12
4	6	COLUMN	2			ROWS	COLUMN	1	2	0.04166667	0.41666667E-01
5	5	ROWS	COLUMN	1	1	ROWS	COLUMN	1	1	0.26388889	0.26388889E 00
5	6	ROWS	COLUMN	1	1	ROWS	COLUMN	1	2	-0.11111111	-0.11111111E 00
6	6	ROWS	COLUMN	1	2	ROWS	COLUMN	1	2	0.26388889	0.26388889E 00

See comment [26]

THE DETERMINANT OF THE CORRELATION MATRIX IS 0.51840000081141 0.51840000081141E 00

Y	1	MU					7.00000000	0.70000000E 01
Y	2	ROWS	1				-1.66666667	-0.16666667E 01
Y	3	COLUMN	1				1.00000000	0.10000000E 01
Y	4	COLUMN	2				-1.00000000	-0.10000000E 01
Y	5	ROWS	COLUMN	1	1		1.66666667	0.16666667E 01
Y	6	ROWS	COLUMN	1	2		1.66666667	0.16666667E 01

LISTING OF CONSTANTS, LEAST-SQUARES MEANS AND STANDARD ERRORS FOR PROBLEM NO. 2

RHM NAME	ROW CODE	INDEPENDENT VARIABLE					NO. OBS.	CONSTANT ESTIMATE	LEAST-SQUARES MEAN	STANDARD ERROR
Y	1	MU					8	7.00000000	7.00000000	0.83333333
Y	2	ROWS		1			4	-1.66666667	5.33333333	1.17851130
Y	0	ROWS		2			4	1.66666667	8.66666667	1.17851130
Y	3	COLUMN		1			3	1.00000000	8.00000000	1.36930639
Y	4	COLUMN		2			3	-1.00000000	6.00000000	1.36930639
Y	0	COLUMN		3			2	-0.00000000	7.00000000	1.58113883
Y	5	ROWS	X	COLUMN	1	1	2	1.66666667	8.00000000	1.58113883
Y	6	ROWS	X	COLUMN	1	2	1	1.66666667	6.00000000	2.23606798
Y	0	ROWS	X	COLUMN	1	3	1	-3.33333333	2.00000000	2.23606798
Y	0	ROWS	X	COLUMN	2	1	1	-1.66666667	8.00000000	2.23606798
Y	0	ROWS	X	COLUMN	2	2	2	-1.66666667	6.00000000	1.58113883
Y	0	ROWS	X	COLUMN	2	3	1	3.33333333	12.00000000	2.23606798

Cell means $\bar{y}_{ij}$. [24]

$\sqrt{\hat{\sigma}^2 \frac{1}{n_{ij}}}$ with $\hat{\sigma}^2 = 5$

$= 2.236^2$

LISTING OF INVERSE ELEMENTS OF SEGMENTS FOR PROBLEM NO. 2

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW				INDEPENDENT VARIABLES COLUMN				INVERSE ELEMENT FIXED POINT FORMAT	INVERSE ELEMENT FLOATING POINT FORMAT
1	1	MU				MU				7.20000000	0.72000000E 01
2	2	ROWS	1			ROWS	1			7.20000000	0.72000000E 01
3	3	COLUMN	1			COLUMN	1			4.60606061	0.46060606E 01
3	4	COLUMN	1			COLUMN	2			1.93939394	0.19393939E 01
4	4	COLUMN	2			COLUMN	2			4.60606061	0.46060606E 01
5	5	ROWS	COLUMN	1	1	ROWS	COLUMN	1	1	4.60606061	0.46060606E 01
5	6	ROWS	COLUMN	1	1	ROWS	COLUMN	1	2	1.93939394	0.19393939E 01
6	6	ROWS	COLUMN	1	2	ROWS	COLUMN	1	2	4.60606061	0.46060606E 01

LEAST-SQUARES ANALYSIS OF VARIANCE

Y

SOURCE	D.F.		SUM OF SQUARES	MEAN SQUARES	F
TOTAL	8	$y'y$	458.000000		
TOTAL REDUCTION	6	$R(\mu,\alpha,\beta,\gamma)$	448.000000	74.666667	14.933
MU-YM	1		352.800000	352.800000	70.560
ROWS	1	SSA_w	20.000000	20.000000	4.000
COLUMN	2	SSB_w	5.333333	2.666667	0.533
ROWS X COLUMN	2	$R(\gamma\|\mu,\alpha,\beta)$	36.363636	18.181818	3.636
REMAINDER	2	SSE	10.000000	5.000000	

[25]

$$\frac{\mu^{o2}}{\text{an inverse element}} = \frac{7^2}{.13888} = \frac{49}{5/36} = \frac{42^2}{5} = 8.4 \times 42 = 352.8$$ [26]

Note the occurrence of SSA_w and SSB_w here – resulting from the computing procedure being used (a procedure that utilizes $\Sigma\alpha_i = 0$ and $\Sigma\beta_j = 0$) and the fact that the data have all cells filled. See paper BU-608-M. [26a]

```
                    LISTING OF PARAMETER CARDS FOR PROBLEM NO.  3

IJOB NAB  NCD  ICN1 NLHM NRHM NMEA NME NNEA  NNE  N2F  NPL  NLC NCPR IRAN MPOP LIOP  IN   FILENAME
  3   0    1    1    5    1    0    2    0    0    1    0    0    0    0    1    3    5

FOR I=   1  MEN(I)  LIT(I)  NCL(I)  MPOL  LHE(I)  IREF(I)
              1      ROWS      2      0      1       1

IDEN(J) WHERE J=K2 BY STEPS OF ONE AS LONG AS J IS LESS THAN K2+NCL(I)+1
   2          1

FOR I=   2  MEN(I)  LIT(I)  NCL(I)  MPOL  LHE(I)  IREF(I)
              2     COLUMS     3      0      1       2

IDEN(J) WHERE J=K2 BY STEPS OF ONE AS LONG AS J IS LESS THAN K2+NCL(I)+1
   1          3          2

FOR I=   1  INT1(I) INT2(I)  NMC(I)
              1       2        1

MSCL(J) WHERE J=   1  INCREMENTED BY ONE UNTIL  J IS GREATER THAN    0+   1
   102

FOR I= 1  NEGY(I) LNY(I) LHY(I) KREG(I) NDECY(I) YM(I) LITY(I)
              0      0      2      3       0    0.00000  Y
```

Problem No. 3

This is an adaptation of an illustration in paper BU-417-M.

	Data			Totals
	2,4,6	4,6	5	27
	12,8	11,7	-	38
Totals	32	28	5	65

$R(\mu) = 422\frac{1}{2}$

$R(\alpha|\mu) = 60$ $\qquad R(\beta|\mu) = 3.3$

$R(\beta|\mu,\alpha) = .318$ $\qquad R(\alpha|\mu,\beta) = 57.018$ [27]

$R(\gamma|\mu,\alpha,\beta) = 2.18$

$SSE = 26$

$SST = 511$

```
               LISTING OF X MATRIX FOR PROBLEM NO.  3
   1.00  -1.00   1.00   0.00  -1.00   2.00
   1.00  -1.00   1.00   0.00  -1.00   4.00
   1.00  -1.00   1.00   0.00  -1.00   6.00
   1.00  -1.00  -1.00  -1.00   1.00   4.00
   1.00  -1.00  -1.00  -1.00   1.00   6.00
   1.00  -1.00   0.00   1.00   0.00   5.00
   1.00   1.00   1.00   0.00   1.00  12.00
   1.00   1.00   1.00   0.00   1.00   8.00
   1.00   1.00  -1.00  -1.00  -1.00  11.00
   1.00   1.00  -1.00  -1.00  -1.00   7.00
```

TOTAL LEAST-SQUARES ANALYSIS. NO EQUATIONS ABSORBED. DF=NO. CARDS= 10

DISTRIBUTION OF CLASS AND SUBCLASS NUMBERS FOR PROBLEM NO. 3

IDENTIFICATION			NO.	
ROWS	2		4	
ROWS	1		6	
COLUMS	1		5	
COLUMS	3		1	
COLUMS	2		4	
ROWS X COLUMS	2	1	2	
ROWS X COLUMS	2	3	0	MISS 1
ROWS X COLUMS	2	2	2	
ROWS X COLUMS	1	1	3	
ROWS X COLUMS	1	3	1	
ROWS X COLUMS	1	2	2	

When there are empty cells, the program does some re-sequencing of rows and columns. [28]

OVERALL MEANS AND STANDARD DEVIATIONS OF LHM FOR PROBLEM NO. 3

CODED LHM	INDEPENDENT VARIABLES				MEAN	S.D.
1	MU				1.00000	0.00000
2	ROWS		2		-0.20000	1.03280
3	COLUMS		1		0.10000	0.99443
4	COLUMS		3		-0.30000	0.67495
5	ROWS	COLUMS	2	1	-0.10000	0.99443

OVERALL MEANS AND STANDARD DEVIATIONS OF RHM

Y MEAN= 6.50000 S.D.= 3.13581

SUMS OF SQUARES, C.P. AND CORRELATIONS AMONG LHM FOR PROBLEM NO. 3

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW				COLUMN				S.SQS. OR C.P.	CORRELATION
1	1	MU				MU				10.00000000	1.0000
1	2	MU				ROWS	2			-2.00000000	0.0000
1	3	MU				COLUMS	1			1.00000000	0.0000
1	4	MU				COLUMS	3			-3.00000000	0.0000
1	5	MU				ROWS	COLUMS	2	1	-1.00000000	0.0000
2	2	ROWS	2			ROWS	2			10.00000000	1.0000
2	3	ROWS	2			COLUMS	1			-1.00000000	-0.0865
2	4	ROWS	2			COLUMS	3			-1.00000000	-0.2550
2	5	ROWS	2			ROWS	COLUMS	2	1	1.00000000	0.0865
3	3	COLUMS	1			COLUMS	1			9.00000000	1.0000
3	4	COLUMS	1			COLUMS	3			4.00000000	0.7118
3	5	COLUMS	1			ROWS	COLUMS	2	1	-1.00000000	-0.1011
4	4	COLUMS	3			COLUMS	3			5.00000000	1.0000
4	5	COLUMS	3			ROWS	COLUMS	2	1	0.00000000	-0.0497
5	5	ROWS	COLUMS	2	1	ROWS	COLUMS	2	1	9.00000000	1.0000

SUMS OF CROSSPRODUCTS AND CORRELATIONS OF LHM WITH RHM FOR PROBLEM NO. 3

RHM	LHM	RHM NAME	INDEPENDENT VARIABLE				C.P.	CORRELATION
1	1	Y	MU				65.00000000	0.0000
1	2	Y	ROWS	2			11.00000000	0.8234
1	3	Y	COLUMS	1			4.00000000	-0.0891
1	4	Y	COLUMS	3			-23.00000000	-0.1837
1	5	Y	ROWS	COLUMS	2	1	0.00000000	0.2316

LISTING OF INVERSE ELEMENTS FOR PROBLEM NO. 3

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW				COLUMN				INVERSE ELEMENT FIXED POINT FORMAT	FLOATING POINT FORMAT
1	1	MU				MU				0.17939815	0.17939815E 00
1	2	MU				ROWS	2			0.04513889	0.45138889E-01
1	3	MU				COLUMS	1			-0.10300926	-0.10300926E 00
1	4	MU				COLUMS	3			0.19907407	0.19907407E 00
1	5	MU				ROWS	COLUMS	2	1	0.00347222	0.34722222E-02
2	2	ROWS	2			ROWS	2			0.11458333	0.11458333E 00
2	3	ROWS	2			COLUMS	1			-0.02430556	-0.24305556E-01
2	4	ROWS	2			COLUMS	3			0.06944444	0.69444444E-01
2	5	ROWS	2			ROWS	COLUMS	2	1	-0.01041667	-0.10416667E-01
3	3	COLUMS	1			COLUMS	1			0.23495370	0.23495370E 00
3	4	COLUMS	1			COLUMS	3			-0.25462963	-0.25462963E 00
3	5	COLUMS	1			ROWS	COLUMS	2	1	0.01736111	0.17361111E-01
4	4	COLUMS	3			COLUMS	3			0.53703704	0.53703704E 00
4	5	COLUMS	3			ROWS	COLUMS	2	1	-0.01388889	-0.13888889E-01
5	5	ROWS	COLUMS	2	1	ROWS	COLUMS	2	1	0.11458333	0.11458333E 00

THE DETERMINANT OF THE CORRELATION MATRIX IS 0.34133333333337504 0.34133333333337504E 00

Y	1	MU				7.16666667	0.71666667E 01
Y	2	ROWS	2			2.50000000	0.25000000E 01
Y	3	COLUMS	1			-0.16666667	-0.16666667E 00
Y	4	COLUMS	3			0.33333333	0.33333333E 00
Y	5	ROWS	COLUMS	2	1	0.50000000	0.50000000E 00

LISTING OF CONSTANTS, LEAST-SQUARES MEANS AND STANDARD ERRORS FOR PROBLEM NO. 3

RHM NAME	ROW CODE	INDEPENDENT VARIABLE					NO. OBS.	CONSTANT ESTIMATE	LEAST-SQUARES MEAN	STANDARD ERROR
Y	1	MU					10	7.16666667	7.16666667	0.96585215
Y	2	ROWS	2				4	2.50000000	9.66666667	1.41355868
Y	0	ROWS	1				6	-2.50000000	4.66666667	1.02920322
Y	3	COLUMS	1				5	-0.16666667	7.00000000	1.04083300
Y	4	COLUMS	3				1	0.33333333	7.50000000	2.40745370
Y	0	COLUMS	2				4	-0.16666667	7.00000000	1.14017542
Y	5	ROWS	X COLUMS	2	1		2	0.50000000	$\boxed{10.00000000}$	$\boxed{1.61245155}$
Y	0	ROWS	X COLUMS	2	2		2	-0.50000000	$\boxed{9.00000000}$	$\boxed{1.61245155}$
Y	0	ROWS	X COLUMS	1	1		3	-0.50000000	$\boxed{4.00000000}$	$\boxed{1.31656118}$
Y	0	ROWS	X COLUMS	1	3		1	0.00000000	$\boxed{5.00000000}$	$\boxed{2.28035085}$
Y	0	ROWS	X COLUMS	1	2		2	0.50000000	$\boxed{5.00000000}$	$\boxed{1.61245155}$

Cell means and values of $\sqrt{\hat{\sigma}^2 \frac{1}{n_{ij}}}$ [29]

LISTING OF INVERSE ELEMENTS OF SEGMENTS FOR PROBLEM NO. 3

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW				COLUMN				INVERSE ELEMENT FIXED POINT FORMAT	FLOATING POINT FORMAT
1	1	MU				MU				5.57419355	0.55741935E 01
2	2	ROWS	2			ROWS	2			8.72727273	0.87272727E 01
3	3	COLUMS	1			COLUMS	1			8.75471698	0.87547170E 01
3	4	COLUMS	1			COLUMS	3			4.15094340	0.41509434E 01
4	4	COLUMS	3			COLUMS	3			3.83018868	0.38301887E 01
5	5	ROWS	COLUMS	2	1	ROWS	COLUMS	2	1	8.72727273	0.87272727E 01

LEAST-SQUARES ANALYSIS OF VARIANCE

Y

SOURCE	D.F.		SUM OF SQUARES	MEAN SQUARES	F
TOTAL	10	$y'y$	511.000000		
TOTAL REDUCTION	5	$R(\mu,\alpha,\beta,\gamma)$	485.000000	97.000000	18.654
MU-YM	1		286.296774	286.296774	55.057
ROWS	1		54.545455	54.545455	10.490
COLUMS	2		0.207547	0.103774	0.020
ROWS X COLUMS	1	$R(\gamma\|\mu,\alpha,\beta)$	2.181818	2.181818	0.420
REMAINDER	5	SSE	26.000000	5.200000	

[30]

$$\frac{\mu^{o2}}{\text{an inverse element}} = \frac{(7.1\dot{6})^2}{.1793} = \frac{(7\frac{1}{6})^2}{.1793} = 285$$ [31]

The terms labeled "Rows" and "Columns" are an enigma. Because of both the computing procedure (which uses $\Sigma\alpha_i = 0$ and $\Sigma\beta_j = 0$) and the empty cells in the data, these terms are not reductions in sums of squares easily calculated as $R = \underset{\sim}{b}^{o\prime}\underset{\sim}{X}'\underset{\sim}{y}$. However, in this small data set we can look at the pattern of filled cells, namely [32]

*	*	*
*	*	

,

and "guess" at what hypothesis "Rows" and "Columns" may represent, and then use $Q = (\underset{\sim}{K}'\underset{\sim}{b}^o - \underset{\sim}{m})'(\underset{\sim}{K}'\underset{\sim}{GK})^{-1}(\underset{\sim}{K}'\underset{\sim}{b}^o - \underset{\sim}{m})$ of $F = Q/s\hat{\sigma}^2$, to check our guess. For example, "Rows" has one degree of freedom and it may be testing

$$H: \mu_{11} + \mu_{12} = \mu_{21} + \mu_{22} \quad \text{for } \mu_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij}\ .$$ [33]

To test this hypothesis we use

$$b^o = (\mu^o_{11}\ \ \mu^o_{12}\ \ \mu^o_{13}\ \ \mu^o_{21}\ \ \mu^o_{22}) = (4\ 5\ 5\ 10\ 9) \text{ with } G = \text{diag}\{\tfrac{1}{3}\ \tfrac{1}{2}\ 1\ \tfrac{1}{2}\ \tfrac{1}{2}\}$$

and so

$$Q = \frac{(4+5-10-9)^2}{\frac{1}{3}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}} = \frac{100}{11/6}\quad\frac{600}{11} = 54.5454,$$

which is the "Rows" entry in the computer ANOVA output. Similarly, it will be found that the "Columns" entry is testing

$$H: \mu_{11} + \mu_{21} = \mu_{12} + \mu_{22}$$
$$\mu_{11} + \mu_{12} = 2\mu_{13}$$

for which

$$\underset{\sim}{K}'\underset{\sim}{b}^{o} = \begin{bmatrix} 1 & -1 & 0 & 1 & -1 \\ 1 & 1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} [illegible] \\ 5 \\ 5 \\ 10 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

and [34]

```
                   LISTING OF PARAMETER CARDS FOR PROBLEM NO.  4

 IJOB NAB  NCD  ICN1 NLHM NRHM NMEA NME NNEA  NAE  N2F  NPR  NLC NCPR IRAN MPOP LIOP  IN     FILENAME
   4   0    1    0    4    1    0    1    0    0    0    1    0    1    0    1    3    5

 FOR I=   1  MEN(I)  LIT(I)  NCL(I)  MPOL  LME(I)  IBEG(I)
                1    LEVEL     3       0      1       1

 IDEN(J) WHERE J=K2 BY STEPS OF ONE AS LONG AS J IS LESS THAN K2+NCL(I)+1
     1        2        3

 FOR K= 1  NEGX(K) LOGE(K) LOC(K) NREGP(K) LETX(K) JBEG(K) NDECX(K) XM(K) LITR(K)
               0       0      1       1        1       2        0    0.00000   CHILD

 FOR I= 1  NEGY(I) LNY(I) LHY(I) KPEF(I) NDECY(I) YM(I) LITY(I)
               0      0      2      3       0     0.00000 INVEST
```

Problem No. 4

This is the covariance example of Table 8.5, page 353 of LM. [35]

y	x	y	x	y	x
74	3	76	2	85	4
68	4	80	4	93	6
77	2				

(continued from above)

$$\underset{\sim}{K}'\underset{\sim}{GK} = \begin{bmatrix} 1 & -1 & 0 & 1 & -1 \\ 1 & 1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{3} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \frac{1}{2} & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \frac{1}{2} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 0 & -2 \\ 1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{pmatrix} \frac{11}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{29}{6} \end{pmatrix}.$$

Hence

$$Q = [0 \quad -1] \begin{bmatrix} \frac{11}{6} & \frac{-1}{6} \\ \frac{-1}{6} & \frac{29}{6} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \frac{(-1)^2 6(11)}{319 - 1} = \frac{11}{53} = .20754$$

which is the "Columns" entry in the computer ANOVA output.

```
                    LISTING OF X MATRIX FOR PROBLEM NO.  4
        1.00    1.00    0.00    3.00   74.00
        1.00    1.00    0.00    4.00   68.00
        1.00    1.00    0.00    2.00   77.00
        1.00    0.00    1.00    2.00   76.00
        1.00    0.00    1.00    4.00   80.00
        1.00   -1.00   -1.00    4.00   85.00
        1.00   -1.00   -1.00    6.00   93.00
```

TOTAL LEAST-SQUARES ANALYSIS. NO EQUATIONS ABSORBED. DF=NO. CARDS= 7

DISTRIBUTION OF CLASS AND SUBCLASS NUMBERS FOR PROBLEM NO. 4

IDENTIFICATION		N
LEVEL	1	3
LEVEL	2	2
LEVEL	3	2

OVERALL MEANS AND STANDARD DEVIATIONS OF LHM FOR PROBLEM NO. 4

CODED LHM	INDEPENDENT VARIABLES	MEAN	S.D.
1	MU	1.00000	0.00000
2	LEVEL 1	0.14286	0.89974
3	LEVEL 2	0.00000	0.81650
4	RGRSN CHILD LINEAR	3.57143	1.39728

Because these values relate to entries in the columns of X, see notes [8] and [9], the last entries here are for the covariate. [36]

OVERALL MEANS AND STANDARD DEVIATIONS OF RHM

INVEST MEAN= 79.00000 S.D.= 8.08290

SUMS OF SQUARES, C.P. AND CORRELATIONS AMONG LHM FOR PROBLEM NO. 4

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW	COLUMN	S.SQS. OR C.P.	CORRELATION
1	1	MU	MU	7.00000000	1.0000
1	2	MU	LEVEL 1	1.00000000	0.2500
1	3	MU	LEVEL 2	0.00000000	0.0000
1	4	MU	RGRSN CHILD LINEAR	0.00000000	0.0000
2	2	LEVEL 1	LEVEL 1	5.00000000	1.0000
2	3	LEVEL 1	LEVEL 2	2.00000000	0.4537
2	4	LEVEL 1	RGRSN CHILD LINEAR	-4.57142857	-0.6060
3	3	LEVEL 2	LEVEL 2	4.00000000	1.0000
3	4	LEVEL 2	RGRSN CHILD LINEAR	-4.00000000	-0.5843
4	4	RGRSN CHILD LINEAR	RGRSN CHILD LINEAR	11.71428571	1.0000

SUMS OF CROSSPRODUCTS AND CORRELATIONS OF LHM WITH RHM FOR PROBLEM NO. 4

RHM	LHM	RHM NAME	INDEPENDENT VARIABLE	C.P.	CORRELATION
1	1	INVEST	MU	553.00000000	0.0000
1	2	INVEST	LEVEL 1	41.00000000	-0.8709
1	3	INVEST	LEVEL 2	-22.00000000	-0.5556
1	4	INVEST	RGRSN CHILD LINEAR	43.00000000	0.6346

SUMS OF SQUARES, C.P. AND CORRELATIONS AMONG RHM FOR PROBLEM NO. 4

ROW	COL	RHM	RHM	S.SQS. OR C.P.	CORRELATION
1	1	INVEST	INVEST	44078.99999999	1.0000

LISTING OF INVERSE ELEMENTS FOR PROBLEM NO. 4

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW	COLUMN	INVERSE ELEMENT FIXED POINT FORMAT	FLOATING POINT FORMAT
1	1	MU	MU	0.14965986	0.14965986E 00
1	2	MU	LEVEL 1	-0.04761905	-0.47619048E-01
1	3	MU	LEVEL 2	0.00793651	0.79365079E-02
1	4	MU	RGRSN CHILD LINEAR	-0.01587302	-0.15873016E-01
2	2	LEVEL 1	LEVEL 1	0.33333333	0.33333333E 00
2	3	LEVEL 1	LEVEL 2	-0.05555556	-0.55555556E-01
2	4	LEVEL 1	RGRSN CHILD LINEAR	0.11111111	0.11111111E 00
3	3	LEVEL 2	LEVEL 2	0.38888889	0.38888889E 00
3	4	LEVEL 2	RGRSN CHILD LINEAR	0.11111111	0.11111111E 00
4	4	RGRSN CHILD LINEAR	RGRSN CHILD LINEAR	0.16666667	0.16666667E 00

THE DETERMINANT OF THE CORRELATION MATRIX IS 0.3951219512309762 0.3951219512309762E 00

INVEST	1 MU	79.95238095	0.79952381E 02	
INVEST	2 LEVEL 1	-6.66666667	-0.66666667E 01	
INVEST	3 LEVEL 2	-1.66666667	-0.16666667E 01	
INVEST	4 RGRSN CHILD LINEAR	0.50000000	0.50000000E 00	$\hat{b} = \frac{1}{2}$, Eq. (42), LM p. 353. [37]

LISTING OF CONSTANTS, LEAST-SQUARES MEANS AND STANDARD ERRORS FOR PROBLEM NO. 4

RHM NAME	ROW CODE	INDEPENDENT VARIABLE	NO. OBS.	CONSTANT ESTIMATE		LEAST-SQUARES MEAN	STANDARD ERROR
INVEST	1	MU	7	79.95238095		79.95238095	2.00356433
INVEST	2	LEVEL 1	3	-6.66666667	add	73.28571429	3.22564133
INVEST	3	LEVEL 2	2	-1.66666667	to	78.28571429	3.85706938
INVEST	0	LEVEL 3	2	8.33333333	zero	88.28571429	4.74801547
INVEST	4	RGRSN CHILD LINEAR		0.50000000			2.11476293

LISTING OF INVERSE ELEMENTS OF SEGMENTS FOR PROBLEM NO. 4

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW	COLUMN	INVERSE ELEMENT FIXED POINT FORMAT	FLOATING POINT FORMAT
1	1	MU	MU	6.68181818	0.66818182E 01
2	2	LEVEL 1	LEVEL 1	3.07317073	0.30731707E 01
2	3	LEVEL 1	LEVEL 2	0.43902439	0.43902439E 00
3	3	LEVEL 2	LEVEL 2	2.63414634	0.26341463E 01
4	4	RGRSN CHILD LINEAR	RGRSN CHILD LINEAR	6.00000000	0.60000000E 01

RESIDUAL MATRICES FOR RIGHT HAND MEMBERS

JOB	ROW	COL	RHM	RHM	ERROR SS OR CP	ERROR MS OR COV	CORRELATION
4	1	1	INVEST	INVEST	80.500000	26.83333349	1.0000

LEAST-SQUARES ANALYSIS OF VARIANCE

INVEST

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARES	F		
TOTAL	7	44779.000000			$y'y$	
TOTAL REDUCTION	4	43099.500000	10999.625000	409.924	$R(\mu,\alpha,b)$	LM p. 354
MU-YM	1	42712.742424	42712.742424	1591.779		
LEVEL	2	153.658537	76.829268	2.863	$R(\alpha\mid\mu,b)$	LM p. 355, Table 8.6b
CHILD F LINEAR	1	1.500000	1.500000	0.056	$R(b\mid\mu,\alpha)$	LM p. 354, Table 8.6a
REMAINDER	3	80.500000	26.833333		SSE	

[38]

$$\frac{\mu^{o2}}{\text{inverse element}} = \frac{79.9^2}{.149} = 42{,}712$$, within rounding error.

Note: No analysis has been made using a different slope for each group. [39]

LISTING OF PARAMETER CARDS FOR PROBLEM NO. 5

```
IJOB NAB  NCD  ICN1 NLHM NRHM NMEA NME NNEA  NNE  N2F  NPP  NLC NCPR IRAN MPOP LIOP  IN     FILENAME
 5    0    1    0    7    1    0    2    0    0    0    1    0    0    0    0    3    5

FOR I=    1  MEN(I)   LIT(I)   NCL(I)  MPOL   LME(I)   IBEG(I)
              1       TREATM    3        0      1        1

IDEN(J) WHERE J=K2 BY STEPS OF ONE AS LONG AS J IS LESS THAN K2+NCL(I)+1
   1          2          3

FOR I=    2  MEN(I)   LIT(I)   NCL(I)  MPOL   LME(I)   IBEG(I)
              2       VARIET    4        0      1        2

IDEN(J) WHERE J=K2 BY STEPS OF ONE AS LONG AS J IS LESS THAN K2+NCL(I)+1
   1          2          3      4

FOR K= 1  NEGX(K) LOCF(K) LQC(K) NREGP(K) LFTX(K) JBEG(K) NDECX(K) XM(K) LITR(K)
             1       0       1       1        1       5       0     0.00000  SEEDPL

FOR I= 1  NEGY(I) LNY(I) IHY(I) KFEC(I) NDECY(I) YM(I) LITY(I)
             1      0      2      3       0     0.00000 WEIGHT
```

y x

8 2 | 12 7 7 3

13 4 | — 11 5

9 3 |

—

6 5 12 6

12 3 14 4 — —

9 6 14 6 10 4

— 7 2 16 8 12 6

1? 5

1? 7

LISTING OF X MATRIX FOR PROBLEM NO. 5

1.00	1.00	0.00	1.00	0.00	0.00	2.00	8.00
1.00	1.00	0.00	1.00	0.00	0.00	4.00	13.00
1.00	1.00	0.00	1.00	0.00	0.00	3.00	9.00
1.00	1.00	0.00	0.00	0.00	1.00	7.00	12.00
1.00	1.00	0.00	-1.00	-1.00	-1.00	3.00	7.00
1.00	1.00	0.00	-1.00	-1.00	-1.00	5.00	11.00
1.00	0.00	1.00	1.00	0.00	0.00	5.00	6.00
1.00	0.00	1.00	1.00	0.00	0.00	3.00	12.00
1.00	0.00	1.00	0.00	1.00	0.00	6.00	12.00
1.00	0.00	1.00	0.00	1.00	0.00	4.00	14.00
1.00	-1.00	-1.00	0.00	1.00	0.00	6.00	9.00
1.00	-1.00	-1.00	0.00	1.00	0.00	2.00	7.00
1.00	-1.00	-1.00	0.00	0.00	1.00	6.00	14.00
1.00	-1.00	-1.00	0.00	0.00	1.00	5.00	16.00
1.00	-1.00	-1.00	-1.00	-1.00	-1.00	4.00	10.00
1.00	-1.00	-1.00	-1.00	-1.00	-1.00	6.00	14.00
1.00	-1.00	-1.00	-1.00	-1.00	-1.00	5.00	11.00
1.00	-1.00	-1.00	-1.00	-1.00	-1.00	7.00	13.00

Problem No. 5

Exercise 8.12, LM p. 375. [40]

This is a covariance analysis for the 2-way crossed classification with interaction.

TOTAL LEAST-SQUARES ANALYSIS. NO EQUATIONS ABSORBED. DF=NO. CARDS= 18

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DISTRIBUTION OF CLASS AND SUBCLASS NUMBERS FOR PROBLEM NO. 5

IDENTIFICATION		NO.
TREATM	1	6
TREATM	2	4
TREATM	3	8
VARIET	1	5
VARIET	2	4
VARIET	3	3
VARIET	4	6

Note: The program was unable [41] to carry out the with interaction analysis. Apparently the program requires data to have at least one row with observations in all columns (and one column with observations in all rows).

OVERALL MEANS AND STANDARD DEVIATIONS OF LHM FOR PROBLEM NO. 5

CODED LHM	INDEPENDENT VARIABLES		MEAN	S.D.
1	MU		1.00000	0.00000
2	TREATM	1	-0.11111	0.90025
3	TREATM	2	-0.22222	0.80845
4	VARIET	1	-0.05556	0.80237
5	VARIET	2	-0.11111	0.75840
6	VARIET	3	-0.16667	0.70711
7	RGRSN	SEEDPL LINEAR	4.77778	1.76754

OVERALL MEANS AND STANDARD DEVIATIONS OF RHM

WEIGHT MEAN= 11.00000 S.D.= 2.84915

SUMS OF SQUARES, C.P. AND CORRELATIONS AMONG LHM FOR PROBLEM NO. 5

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW		INDEPENDENT VARIABLES COLUMN		S.SQS. OR C.P.	CORRELATION
1	1	MU		MU		18.00000000	1.0000
1	2	MU		TREATM	1	-2.00000000	0.0000
1	3	MU		TREATM	2	-4.00000000	0.0000
1	4	MU		VARIET	1	-1.00000000	0.0000
1	5	MU		VARIET	2	-2.00000000	0.0000
1	6	MU		VARIET	3	-3.00000000	0.0000
1	7	MU		RGRSN	SEEDPL LINEAR	0.00000000	0.0000
2	2	TREATM	1	TREATM	1	14.00000000	1.0000
2	3	TREATM	1	TREATM	2	8.00000000	0.6107
2	4	TREATM	1	VARIET	1	5.00000000	0.3981
2	5	TREATM	1	VARIET	2	0.00000000	-0.0191
2	6	TREATM	1	VARIET	3	1.00000000	0.0616
2	7	TREATM	1	RGRSN	SEEDPL LINEAR	-10.44444444	-0.3861
3	3	TREATM	2	TREATM	2	12.00000000	1.0000
3	4	TREATM	2	VARIET	1	6.00000000	0.5239
3	5	TREATM	2	VARIET	2	4.00000000	0.3411
3	6	TREATM	2	VARIET	3	2.00000000	0.1372
3	7	TREATM	2	RGRSN	SEEDPL LINEAR	-6.88888889	-0.2836
4	4	VARIET	1	VARIET	1	11.00000000	1.0000
4	5	VARIET	1	VARIET	2	6.00000000	0.5693
4	6	VARIET	1	VARIET	3	6.00000000	0.6068
4	7	VARIET	1	RGRSN	SEEDPL LINEAR	-8.22222222	-0.3410
5	5	VARIET	2	VARIET	2	10.00000000	1.0000
5	6	VARIET	2	VARIET	3	6.00000000	0.6216
5	7	VARIET	2	RGRSN	SEEDPL LINEAR	-2.44444444	-0.1073
6	6	VARIET	3	VARIET	3	9.00000000	1.0000
6	7	VARIET	3	RGRSN	SEEDPL LINEAR	5.33333333	0.2510
7	7	RGRSN	SEEDPL LINEAR	RGRSN	SEEDPL LINEAR	53.11111111	1.0000

SUMS OF CROSSPRODUCTS AND CORRELATIONS OF LHM WITH RHM FOR PROBLEM NO. 5

RHM	LHM	RHM NAME	INDEPENDENT VARIABLE	C.P.	CORRELATION
1	1	WEIGHT	MU	199.00000000	0.0000
1	2	WEIGHT	TREATM 1	-34.00000000	-0.2752
1	3	WEIGHT	TREATM 2	-50.00000000	-0.1532
1	4	WEIGHT	VARIET 1	-18.00000000	-0.1801
1	5	WEIGHT	VARIET 2	-24.00000000	-0.0544
1	6	WEIGHT	VARIET 3	-24.00000000	0.2628
1	7	WEIGHT	RGRSN SEEDPL LINEAR	53.00000000	0.6191

LISTING OF INVERSE ELEMENTS FOR PROBLEM NO. 5

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW	COLUMN	INVERSE ELEMENT FIXED POINT FORMAT	FLOATING POINT FORMAT
1	1	MU	MU	0.06944219	0.69442188E-01
1	2	MU	TREATM 1	-0.00917804	-0.91780377E-02
1	3	MU	TREATM 2	0.04020468	0.40204678E-01
1	4	MU	VARIET 1	-0.04389349	-0.43893491E-01
1	5	MU	VARIET 2	-0.01432884	-0.14328839E-01
1	6	MU	VARIET 3	0.06001597	0.60015974E-01
1	7	MU	RGRSN SEEDPL LINEAR	-0.01007147	-0.10071475E-01
2	2	TREATM 1	TREATM 1	0.16374269	0.16374269E 00
2	3	TREATM 1	TREATM 2	-0.11403509	-0.11403509E 00
2	4	TREATM 1	VARIET 1	-0.00316764	-0.31676413E-02
2	5	TREATM 1	VARIET 2	0.09917154	0.99171540E-01
2	6	TREATM 1	VARIET 3	-0.07724172	-0.77241715E-01
2	7	TREATM 1	RGRSN SEEDPL LINEAR	0.02923977	0.29239766E-01
3	3	TREATM 2	TREATM 2	0.21929825	0.21929825E 00
3	4	TREATM 2	VARIET 1	-0.09576023	-0.95760234E-01
3	5	TREATM 2	VARIET 2	-0.10453216	-0.10453216E 00
3	6	TREATM 2	VARIET 3	0.12646199	0.12646199E 00
3	7	TREATM 2	RGRSN SEEDPL LINEAR	-0.02631579	-0.26315789E-01
4	4	VARIET 1	VARIET 1	0.31601283	0.31601283E 00
4	5	VARIET 1	VARIET 2	-0.00367528	-0.36752762E-02
4	6	VARIET 1	VARIET 3	-0.23645630	-0.23645630E 00
4	7	VARIET 1	RGRSN SEEDPL LINEAR	0.05945419	0.59454191E-01
5	5	VARIET 2	VARIET 2	0.26143194	0.26143194E 00
5	6	VARIET 2	VARIET 3	-0.18577404	-0.18577404E 00
5	7	VARIET 2	RGRSN SEEDPL LINEAR	0.03606238	0.36062378E-01
6	6	VARIET 3	VARIET 3	0.44564246	0.44564246E 00
6	7	VARIET 3	RGRSN SEEDPL LINEAR	-0.08869396	-0.88693957E-01
7	7	RGRSN SEEDPL LINEAR	RGRSN SEEDPL LINEAR	0.04093567	0.40935673E-01

THE DETERMINANT OF THE CORRELATION MATRIX IS 0.0446014236836163 0.44601423683616320E-01

WEIGHT	1	MU	11.21117609	0.11211176E 02
WEIGHT	2	TREATM 1	-0.60233918	-0.60233918E 00
WEIGHT	3	TREATM 2	0.67543860	0.67543860E 00
WEIGHT	4	VARIET 1	-0.56920078	-0.56920078E 00
WEIGHT	5	VARIET 2	-0.82066277	-0.82066277E 00
WEIGHT	6	VARIET 3	1.50487329	0.15048733E 01
WEIGHT	7	RGRSN SEEDPL LINEAR	0.69005848	0.69005848E 00

LISTING OF CONSTANTS, LEAST-SQUARES MEANS AND STANDARD ERRORS FOR PROBLEM NO. 5

RHM NAME	ROW CODE	INDEPENDENT VARIABLE		NO. OBS.	CONSTANT ESTIMATE	LEAST-SQUARES MEAN	STANDARD ERROR
WEIGHT	1	MU		18	11.21117609	11.21117609	0.70656777
WEIGHT	2	TREATM	1	6	-0.60233918	10.60883691	1.24276319
WEIGHT	3	TREATM	2	4	0.67543860	11.88661469	1.62908385
WEIGHT	0	TREATM	3	8	-0.07309942	11.13807667	1.08039161
WEIGHT	4	VARIET	1	5	-0.56920078	10.64197531	1.46267841
WEIGHT	5	VARIET	2	4	-0.82066277	10.39051332	1.47401252
WEIGHT	6	VARIET	3	3	1.50487329	12.71604938	2.17682452
WEIGHT	[illegible]	VARIET	4	6	-0.11500975	11.09616634	1.30567833
WEIGHT	7	RGRSN SEEDPL LINEAR			0.69005848		0.54249169

LISTING OF INVERSE ELEMENTS OF SEGMENTS FOR PROBLEM NO. 5

ROW CODE	COL CODE	INDEPENDENT VARIABLES ROW		INDEPENDENT VARIABLES COLUMN		INVERSE ELEMENT FIXED POINT FORMAT	INVERSE ELEMENT FLOATING POINT FORMAT
1	1	MU		MU		14.40046785	0.14400468E 02
2	2	TREATM	1	TREATM	1	9.57446809	0.95744681E 01
2	3	TREATM	1	TREATM	2	4.97872340	0.49787234E 01
3	3	TREATM	2	TREATM	2	7.14893617	0.71489362E 01
4	4	VARIET	1	VARIET	1	7.47777778	0.74777778E 01
4	5	VARIET	1	VARIET	2	4.15555556	0.41555556E 01
4	6	VARIET	1	VARIET	3	5.70000000	0.57000000E 01
5	5	VARIET	2	VARIET	2	7.74444444	0.77444444E 01
5	6	VARIET	2	VARIET	3	5.43333333	0.54333333E 01
6	6	VARIET	3	VARIET	3	7.53333333	0.75333333E 01
7	7	RGRSN SEEDPL LINEAR		RGRSN SEEDPL LINEAR		24.42857143	0.24428571E 02

LEAST-SQUARES ANALYSIS OF VARIANCE

WEIGHT

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARES	F
TOTAL	18 y'y p.150	2316.000000		
TOTAL REDUCTION	[illegible]	2236.918129	319.559733	44.450
MU-YM	1	1810.001562	1810.001562	251.765
TREATM	2	2.684086	1.342043	0.187
VARIET	3	5.395906	1.798635	0.250
SEEDPL B LINEAR	1	11.632414	11.632414	1.618
REMAINDER	11	79.081871	7.189261	

This is the analysis without interaction and so is different from the analysis on pages 147-152 of the Solutions Manual. [42]